Teorema da Divergência.

$$\iiint_D \underbrace{\operatorname{div} F} = \boxed{\iint_{\partial D} F \cdot N_{ext}}$$ FLUXO

$D \subset \mathbb{R}^3$, aberto, limitado, regular

$F: \mathbb{R}^3 \to \mathbb{R}^3$, C^1 em D.

$$\operatorname{div} F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}.$$

Exemplo: $F(x,y) = (x, y)$

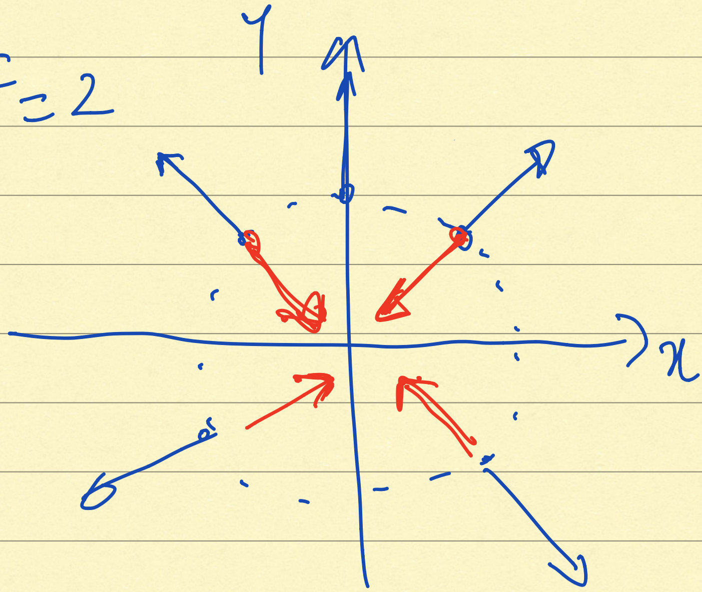
$$\operatorname{div} F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 1 + 1 = \underline{2}$$

$\operatorname{div} F = 2$

$F(x,y) = (x,y)$

$G(x,y) = (-x,-y)$

$\operatorname{div} G = -2$



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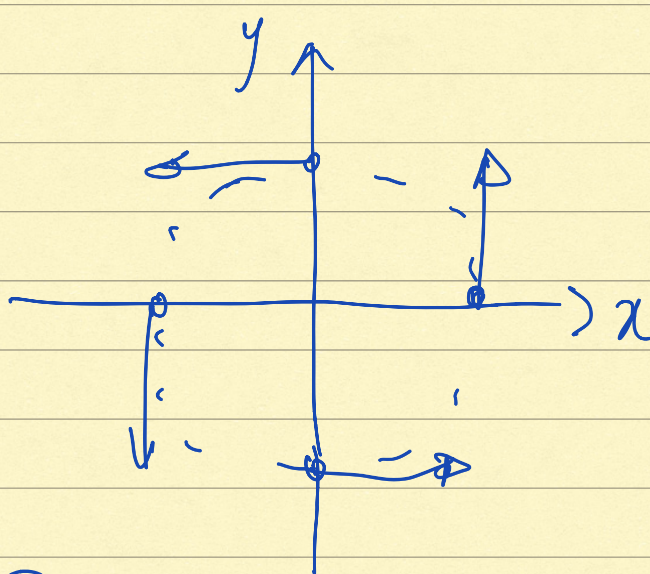
2) $F(x,y) = (-y, x)$

$F(1,0) = (0,1)$



$\operatorname{div} F = 0$

Exemplo: $F(x,y,z) = (-y, x, 1)$ ✓

$S: z = x^2 + y^2 < 1.$

$\boxed{\iint_S F \cdot N \,??}$ $\boxed{N_z > 0}$ (usando o T. de div.)

$$\iiint_D \operatorname{div} F = \iint_{\partial D} F \cdot N_{ext}$$

???

$\boxed{S \subseteq \partial D}$

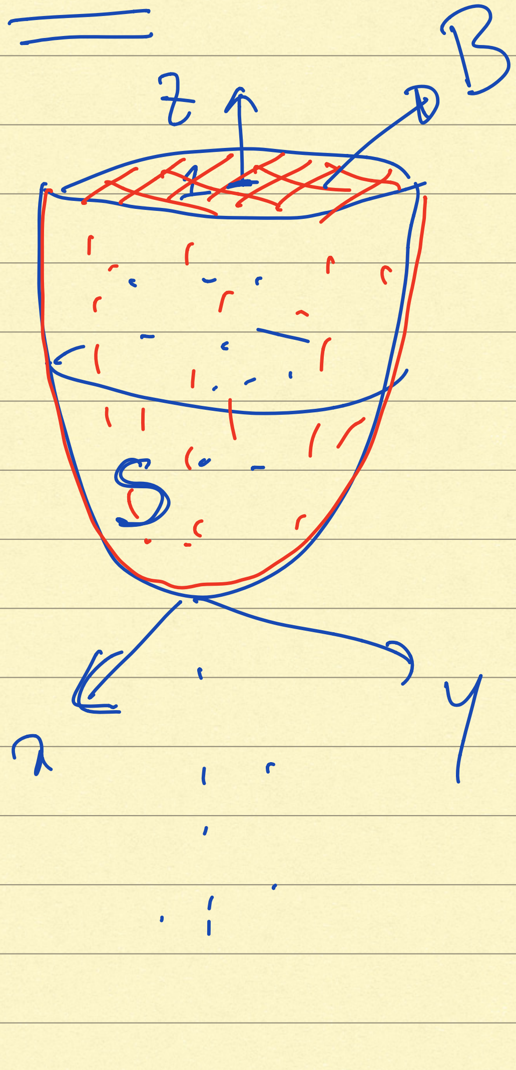
"chave para usar o teorema":

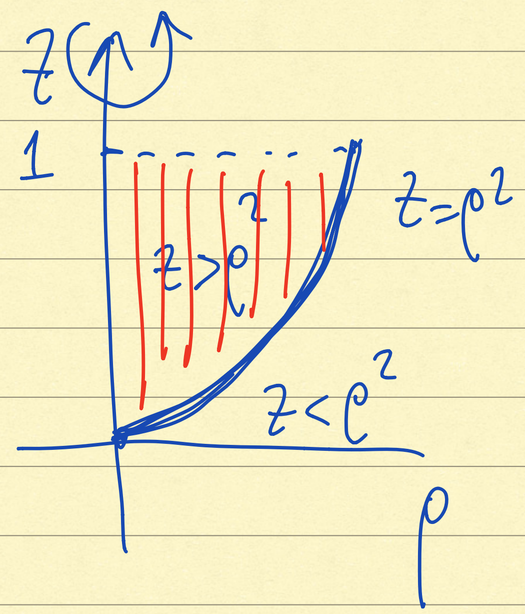
A partir de S construir D tal que $S \subseteq \partial D$.

$S: \quad z = x^2+y^2 < 1.$ $\quad =$

$D: \quad z > x^2+y^2 \; ; \; x^2+y^2<1$ $\quad \geq$ ou $\leq$

limitado





$D: \quad x^2+y^2 < z < 1 \quad$ (sólido)

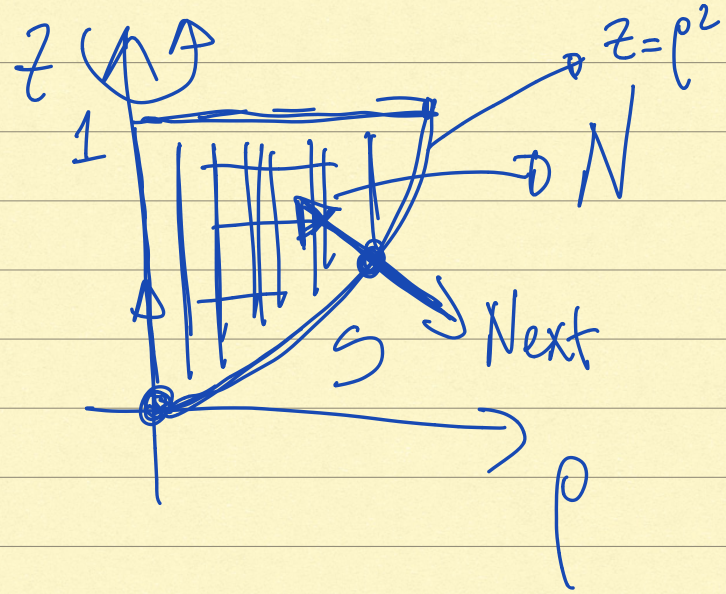
$\partial D: \{x^2+y^2 = z < 1\} \cup \{x^2+y^2 < z;\ z=1\}$

$\partial D = S \cup B$

$$\boxed{\iiint_D \operatorname{div} F} = \boxed{\iint_S F\cdot N_{ext}} + \boxed{\iint_B F\cdot N_{ext}}$$

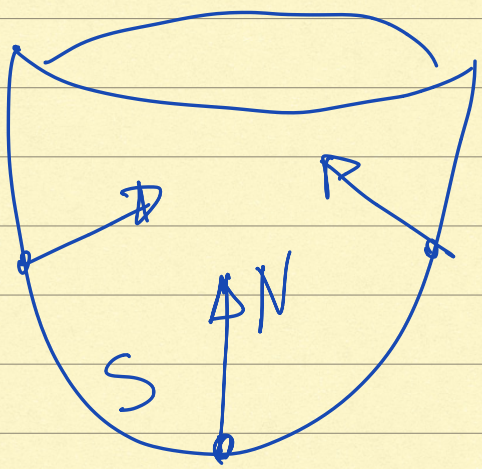
Calcular ¿¿¿ ??? Calcular

$$\iint_S F\cdot N_{ext} = \iiint_D \operatorname{div} F \;\checkmark - \iint_B F\cdot N_{ext}$$



$N_z > 0$

$$\boxed{N = -N_{ext}}$$

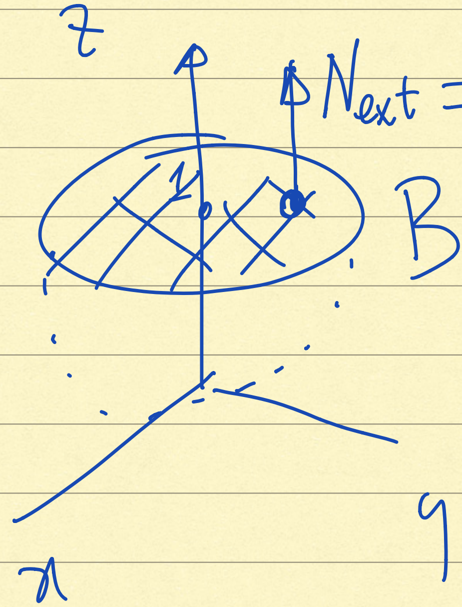


$F(x,y,z) = (-y, x, 1)$

$$\operatorname{div} F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 0$$

$$\iiint_D \operatorname{div} F = 0 .$$

B: $\boxed{z = 1}$; $\boxed{x^2+y^2 < 1}$. plano horizontal



$N_{ext} = (0,0,1)$

$F(x,y,1) = (-y,x,1)$

em B:

$$F \cdot N_{ext} = (-y,x,1)\cdot(0,0,1) = 1$$

$$\iint_B F\cdot N_{ext} = \iint_B 1 = \mathrm{Vol}_2(B) = \pi$$

$x^2+y^2<1$

círculo de raio 1.

$$\iint_S F \cdot N_{ext} = 0 - \pi$$

$$N = -N_{ext}$$

$$\boxed{\begin{array}{l} \displaystyle\iint_S F \cdot N = -\iint_S F \cdot N_{ext} = \pi \\ N_z > 0 \end{array}}$$

✓